The XXZ spin-$1/2$ chain: an integrable approach

Karol K. Kozlowski [1]

Univ Lyon, ENS de Lyon, Univ Claude Bernard Lyon 1, CNRS, Laboratoire de Physique, F-69342 Lyon, France

Quantum integrability of a model refers to the presence of an underlying algebraic structure, called quantum group, which allows one to compute explicitly many physically interesting observable which, usually, are absolutely out of reach. These lectures aim at introducing several of the modern techniques of quantum integrability on the paradigmatic example of the XXZ spin-$1/2$ chain, which is probably nowadays the best understood quantum integrable systems. After a brief review of the physical origin of the model, we will introduce the so-called Algebraic Bethe Ansatz method which allows one to build the Eigenvectors and Eigenvalues of the XXZ spin-$1/2$ Hamiltonian. In this approach, the spectral quantities are parametrised in terms of solutions to a set of coupled algebraic equations called Bethe Ansatz equations. We shall study these equations and explain how one can extract various physically pertinent pieces of information out of them, such as the per site ground state energy, the structure of excitation along and, in particular, access to the universal structure of the spectrum.

In a second part of the lecture, we shall connect, through the transfer matrix approach, the XXZ chain with the celebrated six-vertex model, a very rich model of two-dimensional statistical mechanics. We shall discuss the phase diagram of that model and compute its partition function for various kinds of boundary conditions. This will lead us to consider domain wall boundary conditions and will allow us to present an elegant proof of the alternating sign matrix enumeration problem.

In a third part of the lectures, we shall explain how one can build on integrability techniques so as to study the physics of the chain at finite temperature. This will lead us to introduce the concept of quantum transfer matrix. After studying its spectrum through the associated system of Bethe Ansatz equations, we shall provide a closed expression for the per-site free energy of the XXZ spin-$1/2$ chain at finite temperature.

Finally, in a fourth part of the lectures, we shall present various techniques that allow one to compute, in fully closed form, the correlation functions of the XXZ chain be it at zero or at finite temperature.

[1] e-mail: karol.kozlowski@ens-lyon.fr